

Bifurcation Diagram

Classroom worksheet · Interactive simulation: <https://lkforge.com/tools/physics/bifurcation-diagram/>

Name: _____ Date: _____

Learning objectives

- Describe the logistic map and its long-term behavior.
- Identify period-doubling as a route to chaos.
- Locate where chaos begins.

Variables to change

- Growth parameter r
- Initial population

Procedure

1. Set a low r and observe the population settle to a single steady value.
2. Increase r and watch it split into 2, then 4, then more values.
3. Increase r past about 3.57 and observe chaotic behavior.

Observations

Record the number of stable values as r increases and where the pattern turns chaotic.

Questions

1. What does each branch split (bifurcation) represent?
2. What is the logistic map equation?
3. Roughly where does chaos begin?
4. Is the chaotic region completely disordered?

5. What does this simple equation demonstrate?

Answer key (instructors)

- **1.** A period doubling — the population now cycles between twice as many values.
- **2.** $x_{n+1} = r \cdot x_n \cdot (1 - x_n)$.
- **3.** Around $r \approx 3.57$, after an infinite cascade of period doublings.
- **4.** No — it contains narrow "windows" of periodic behavior (e.g. period-3) amid the chaos.
- **5.** That very simple deterministic rules can produce complex, chaotic behavior.

The logistic map $x_{n+1} = r \cdot x_n \cdot (1 - x_n)$ settles to one value at low r , then period-doubles (2, 4, 8, ...) as r rises, reaching chaos near $r \approx 3.57$. This simple rule shows how deterministic systems can become chaotic through period doubling.